

# Traveling-Wave Reactors: A Truly Sustainable and Full-Scale Resource for Global Energy Needs

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**Abstract** – *Rising environmental and economic concerns have signaled a desire to reduce dependence on hydrocarbon fuels. These concerns have brought the world to an inflection point and decisions made today will dictate what the global energy landscape will look like for the next half century or more. An optimal energy technology for the future must meet stricter standards than in the past; in addition to being economically attractive, it now must also be environmentally benign, sustainable and scalable to global use. For stationary energy, only one existing resource comes close to fitting all of the societal requirements for an optimal energy source: nuclear energy. Its demonstrated economic performance, power density, and emissions-free benefits significantly elevate nuclear electricity generation above other energy sources. However, the current nuclear fuel cycle has some attributes that make it challenging to expand on a global scale.*

*Traveling-wave reactor (TWR) technology, being developed by TerraPower, LLC, represents a potential solution to these limitations by offering a nuclear energy resource which is truly sustainable at full global scale for the indefinite future and is deployable in the near-term. TWRs are capable of offering a ~40-fold gain in fuel utilization efficiency compared to conventional light-water reactors burning enriched fuel. Such high fuel efficiency, combined with an ability to use uranium recovered from river water or sea-water (which has been recently demonstrated to be technically and economically feasible) suggests that enough fuel is readily available for TWRs to generate electricity for 10 billion people at United States per capita levels for million-year time-scales. Interestingly, the Earth's rivers carry into the ocean a flux of uranium several times greater than that required to replace the implied rate-of-consumption, so that the Earth's slowly-eroding crust will provide a readily-accessible flow of uranium sufficient for all of mankind's anticipated energy needs for as long as the sun shines and the rain falls. Moreover, TWRs can naturally retain their efficiently-expended fuel for century length time-scales, so that they intrinsically pose minimal safety and security transportation hazards in addition to being full-scale carbon-free energy sources.*

*This paper describes how TWRs could help move the global energy economy to a more sustainable footing. An economic case and potential impacts on the global energy system are explored. The paper also provides an overview of the practical engineering embodiment of the TWR, new computational tools we have developed for modeling TWRs, the degradation of the plutonium vector in used fuel from TWRs and advanced technological options for repurposing fuel to extract more of its potential energy.*

## I. INTRODUCTION

Fast reactors have been designed, built and operated since the early days of the nuclear industry. Most of these fast reactors used a closed fuel cycle — that is, their used fuel was reprocessed to remove plutonium and other isotopes for reuse — because the

predominant belief in the 1950s was that the world was running out of uranium. The concern about uranium shortages spurred interest in deploying fast reactors — preferably designs offering as high a breeding rate as possible — to produce new fissile plutonium fuel from fertile uranium.

The discovery of large uranium deposits obviated the need for high breeding rates. Meanwhile, concerns about economics, waste, and the proliferation of nuclear weapons technology caused fast reactor development to slow. The prior paradigm, which assumed that fast reactors must operate within a closed fuel cycle, persisted. Heightened concerns about the risks of proliferation from reprocessing led to more recent development programs such as the Advanced Fuel Cycle Initiative (AFCI) and later the Global Nuclear Energy Partnership (GNEP), which sought to reduce that risk. Both programs focused on a strategy of combining actinide streams. Although this fuel cycle strategy does lower proliferation risks, it does not allow for the simplification and lower cost necessary to compete economically with existing LWRs, which use an open fuel cycle.

There is another technology pathway for fast reactors, one that does not require reprocessing facilities and offers an order of magnitude higher fuel efficiency than LWRs. This pathway leads to a so-called breed-and-burn fast reactor operating with an open (once-through) fuel cycle. It thus realizes most of the benefits of a closed fuel cycle without any of the associated costs.

The first known proposal of a fast reactor design that could use an open fuel cycle was made in 1958 by Feinberg<sup>1</sup> who suggested that a breed-and-burn fast reactor could use only natural uranium or depleted uranium as fuel. Other similar concepts were proposed by Driscoll in 1979,<sup>2</sup> Feoktistov in 1988,<sup>3</sup> Teller in 1995,<sup>4</sup> and van Dam in 2000.<sup>5</sup> More recently, Fomin<sup>6</sup> has completed work on the mathematical treatment of the space-dependant criticality in nuclear-burning waves and Sekimoto<sup>7</sup> has made great progress in demonstrating the strengths of this type of reactor. In 2006, TerraPower launched an effort to develop the first practical engineering embodiment of a breed-and-burn fast reactor, producing a design concept now known as a traveling-wave reactor or TWR.<sup>8</sup> TWR designs are being developed for both low- to medium-power (~300-MW<sub>e</sub>) and large power (~1000-MW<sub>e</sub>) applications.

## II. RESULTS

### II.A. Sustainability of a TWR Economy

The main difference between thermal reactors and fast reactors is the degree to which uranium can be burned. Natural uranium, as it is mined, consists of 0.7% U<sup>235</sup> and 99.3% U<sup>238</sup>. Thermal reactors burn primarily U<sup>235</sup>, and are able to convert only modest fractions of U<sup>238</sup> to Pu<sup>239</sup> before their neutron

economies become marginal. As a result, even the best LWRs are able to fission only 0.7%<sup>a</sup> of all uranium that is mined. Mixed-oxide (MOX) recycling can improve this use efficiency by about 30%.

In contrast, fast reactors convert U<sup>238</sup> to fissile Pu<sup>239</sup> or fission U<sup>238</sup> directly. Fast reactors can also be designed to create significantly more fissile fuel than is used. Because of these abilities, fast reactors are able, in principle, to fission essentially all uranium, as it is mined, provided that the fission products (which parasitically absorb neutrons and thereby progressively degrade the reactor's neutron economy) are removed at least once. Even if fission products are never chemically removed from the reactor, it can be designed to fission about 50% of the natural or depleted uranium before its fuel becomes "effectively spent," i.e., no longer capable of producing sufficient neutrons to sustain nuclear reactions.<sup>9,10</sup> One example of a fast reactor design that offers such high-performance breeding capability is a TerraPower TWR cooled by liquid sodium. This reactor is capable of sustaining energy-producing fission when fueled primarily with natural or depleted uranium. Only a small amount of enrichment is needed to start fission going, and no chemical reprocessing of spent fuel is ever required. TWRs of this kind should be able to achieve a fuel utilization efficiency about 40 times that of current LWRs. Such a dramatic increase in fuel efficiency has important implications for the sustainability of global uranium resources.

Uranium is currently mined and extracted from comparatively high-grade terrestrial ores. Uranium sells for roughly \$50 per pound of U<sub>3</sub>O<sub>8</sub>, (\$130 per kilogram of uranium).<sup>11</sup> In a light-water reactor, this amounts to an electricity generation cost of about \$0.0025 per kW<sub>e</sub>-hr which is roughly 5% of the total cost of nuclear electricity. This already low figure shows how relatively insensitive the existing LWR nuclear electricity industry is to changing fuel prices. The exceptionally high fuel utilization of TWRs,

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<sup>a</sup> Generally at most 60% of the 0.7% U<sup>235</sup> in as-mined uranium is recovered via standard commercial isotopic enrichment processes (~0.4% of the as-mined material); the rest of 'fresh' LWR fuel is comprised of the U<sup>238</sup> from the as-mined uranium. Thus, the apparent utilization of as-mined uranium is 1:250. However, some of the U<sup>238</sup> is bred into Pu<sup>239</sup> during the fuel's burn-up in the reactor, amounting to 60-70% of the U<sup>235</sup> burned. The burning of a fraction of this raises the first-pass total uranium utilization to slightly better than 1:200 during this first-pass, and may bring it to not much more than 1:100 with multiple-pass reprocessing and recycling of the Pu<sup>239</sup> recovered in each reprocessing cycle back into "Mixed-Oxide (MOX) fuel" depending on details of reactor design and operation and reprocessing efficiencies.

however, could change this situation in a qualitative way. Because a TWR requires about one-fiftieth the uranium needed by an LWR to produce a given amount of electrical energy, a TWR would have a uranium cost of less than \$0.00004 per kW<sub>e</sub>-hr, which for all practical purposes can be considered negligible. Even if uranium prices increased by a factor of ten, the cost of nuclear electricity produced by a TWR would increase by less than 1%.

This fundamental economic difference between TWRs and LWRs is important because on long timescales, there will be a limited amount of uranium that can be extracted at low cost. As shown in Table I, the known and inferred uranium resources recoverable at a price of ~\$130 or less per kilogram of uranium metal is estimated at 5.5 million metric tons worldwide.<sup>12</sup> Given uranium's natural abundance of approximately 3 ppm in the earth's crust (making uranium roughly as common as tin or zinc), however, that estimate is clearly enormously conservative. Geology still possesses extremely limited knowledge about uranium deposits within the earth's crust.<sup>13</sup>

TABLE I

Known Reserves and Resources for TWR Fuel<sup>12, 14, 15</sup>

Fuel Source	Reserve Size (MT)
Global Depleted Uranium as of 2009	1,500,000
Global LWR Used Fuel as of 2009	270,000
Known Global Uranium Reserves	5,500,000
Estimated Global Uranium Phosphorite Deposits	30,000,000
Estimated Global Uranium Seawater Deposits	4,000,000,000

Nevertheless, once the production price of conventional uranium resources rises sufficiently, other uranium resources such as low-grade ores—and perhaps even uranium recovery from seawater—will become economically viable. TWRs, being largely insensitive to uranium prices, could take advantage of un-conventional uranium resources without a substantial economic penalty. This is significant because these lower-grade resources are available in much greater quantities; the distribution of uranium resources has been found to follow a log-normal distribution, with the quantity of available uranium increasing 300-fold for every 10-fold decrease in ore-concentration.<sup>16</sup> Thus, at the 3 ppm uranium concentration in the earth's crust, the log-normal law implies a ~90,000 times greater amount of uranium available in the crust than from uranium ore deposits that are currently mined.

Most notable for the very long term is the prospect of extracting uranium from seawater, because there is an essentially limitless supply of

uranium dissolved in seawater and it is continually replenished by continental runoff. Uranium is dissolved in seawater at a low concentration, just 3.3 micrograms per liter.<sup>17</sup> Nevertheless, there is such a vast volume of ocean water (~1.3 billion cubic kilometers) that the total amount of uranium dissolved in the seas is enormous: over four billion metric tons, close to 1,000 times the amount contained in conventional terrestrial resources.

Practical technology to extract uranium from seawater has already been demonstrated and the cost of uranium extraction with this current adsorbent technology is estimated at roughly \$96/lb-U<sub>3</sub>O<sub>8</sub>.<sup>18</sup> This value is only about twice the current market price of U<sub>3</sub>O<sub>8</sub>.<sup>19</sup> Economic extraction of uranium from seawater may thus be economically attractive in the not too distant future.

The energy value present in marine reserves of uranium is immense, as Cohen<sup>20</sup> observed long ago. With a 45% efficiency of utilization in TWRs, the extant marine reserve would be able to supply all of the world's present electricity usage for about 1,000,000 years. Even if the world's population grew to 10 billion people, all at per capita electricity usage rates as high as in the United States today, the marine reserves of uranium would supply this scale of a TWR nuclear energy economy for 130,000 years, roughly two dozen times as long as all of recorded history and extending over the entire expected duration of the next Ice Age.

In addition to being an enormous existing store of recoverable energy in immediately useful form, the 'stock' of uranium in the world's oceans is being constantly replenished. Wind and rain constantly erode the Earth's crust, which, as noted above, contains an average uranium concentration of 3 parts per million. Rivers then carry this rain-dissolved crustal uranium into the oceans at a present rate of approximately 10,000 tonnes per year.<sup>21</sup> This is a rate sufficient to meet the world's entire electricity demand, again all at the present-day American level of electrification, without chemical reprocessing when employed in maximum-efficiency TWRs. Meanwhile, the crustal fraction eroded by rivers is constantly replaced by new layers of rock being pushed upward by plate tectonic processes.<sup>22</sup> The inventory of uranium in the Earth's crust is effectively inexhaustible, of the order of 40,000,000,000,000 metric tons — 10,000 times more than is present in the oceans — and could satisfy present-day world energy demands for over a billion years. Through natural geological and meteorological processes, this supply of uranium is unceasingly being made readily available for recovery in the world's oceans, making uranium derived from seawater and efficiently used in TWRs

a truly sustainable energy resource which is continually and naturally renewed.

### *II.B. TWR Engineering Embodiment*

The practical engineering embodiment of a TWR, shown in Figure 1, is based on elements of sodium cooled, fast reactor technology that have been thoroughly tested in a large number of one-of-a-kind reactors over the last fifty years.<sup>23</sup> It consists of a cylindrical reactor core submerged in a large sodium pool in the reactor vessel, which is surrounded by a containment vessel that prevents loss of sodium coolant in case of an unlikely leak from the reactor vessel. The pumps circulate primary sodium coolant through the reactor core exiting at the top and passing through intermediate heat exchangers located in the pool. These heat exchangers have non-radioactive intermediate sodium coolant on the other side of the heat exchanger. Heated intermediate sodium coolant is circulated to the steam generators (not shown) that generate steam to drive turbine and electrical generators. During periods of reactor shut down, the plant electrical loads are provided by the grid and decay heat removal is provided by pony motors on the coolant pumps delivering reduced flow through the heat transport systems. In the event that grid power is not available, decay heat is removed using two dedicated safety class decay heat removal systems: the Reactor Vessel Air Cooling System (RVACS) and the Auxiliary Cooling System (ACS), which operate entirely by natural circulation with no need for electrical power. Finally, reactor containment is formed by an underground containment vessel with an upper steel dome appropriate for beyond design basis accidents in a pool type liquid metal reactor. The TWR arrangement appears similar to other proposed fast reactor designs,<sup>24</sup> but has enhanced features in the RVACS and ACS for better aircraft protection and in heat exchanger design for more effective use of space and increased efficiency. Since the deviation in design from what has been previously built adds additional licensing time, TerraPower purposefully maintained the plant arrangement as traditional as possible so that the innovation could be focused on where it really counts, in the core.

The major distinguishing feature of the TWR from other fast reactor designs is its core. The design is the result of an extensive pre-conceptual study that evaluated various core configurations and compositions. What emerged from these studies was an approximate cylindrical core geometry composed of hexagonally shaped fuel bundles, or assemblies, containing a combination of enriched and depleted uranium metal alloy fuel pins clad in ferritic-

martensitic steel tubes. This core provides a special class of TWR core design where the breed-burn wave does not move through fixed core material. Instead, a “standing” wave of breeding and burning is established by periodically moving core material in and out of the breed-burn region. This movement of fuel assemblies is referred to as “fuel shuffling” and will be described in more detail later. Metal fuel was selected because it offers high heavy metal loadings and excellent neutron economy, which is critical for an effective breed and burn process in TWRs. The uranium metal is alloyed with 5 to 8% zirconium to dimensionally stabilize the alloy during irradiation and to inhibit low-temperature eutectic and corrosion damage of the cladding. A sodium thermal bond fills the gap that exists between the uranium alloy fuel and the inner wall of the clad tube to allow for fuel swelling and to provide efficient heat transfer which keeps the fuel temperatures low. Individual fuel pins have a thin wire from 0.8 to about 1.6 mm diameter helically wrapped around the circumference of the clad tubing to provide coolant space and mechanical separation of individual pins within the hexagonal fuel assembly housing that also serves as the coolant duct. The cladding, wire wrap and housing are fabricated from ferritic-martensitic steel because of its superior irradiation performance as indicated by a significant body of empirical data.<sup>25</sup>

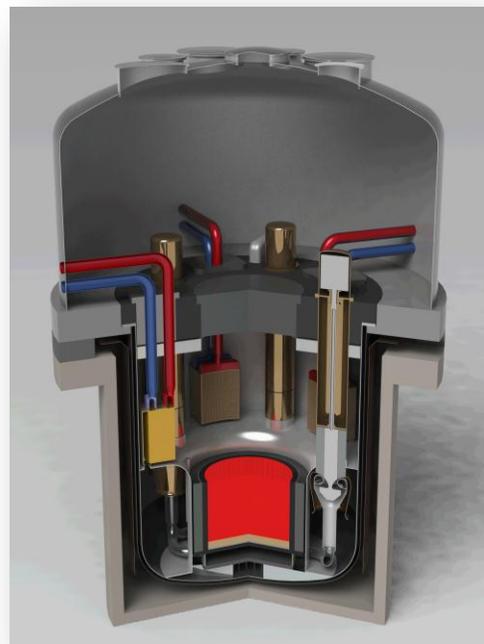


Fig. 1. Possible practical engineering embodiment of a TWR.

Fuel assemblies are clustered together with approximately 5 mm spacing between the flats of the hexagonal ducts in a symmetric mixture of fuel assemblies with enriched and depleted uranium alloy fuel pins. The core contains two types of assemblies – standard assemblies having depleted uranium pins for breeding (fertile assemblies) and a sufficient number of fissile assemblies having fuel pins with uranium enriched (less than 20%) in the  $^{235}\text{U}$  isotope to produce initial criticality and sufficient plutonium breeding to approach a steady state reactor core breed-and-burn condition. The fissile assemblies are primarily located in the central core zone, designated the Active Control Zone (ACZ) shown in orange in Figure 2, which generates most of the core power. Fertile assemblies are primarily placed in the core peripheral region, called the Fixed Control Zone (FCZ) shown in green in Figure 2 and their number is selected such that reactor operation is possible for at least 40 years without the need to bring new fuel into the reactor. In addition, the FCZ also contains a sufficient number of spare fissile and fertile fuel assemblies in the case that replacement assemblies are needed for failed fuel pins. The initial core loading is configured to produce criticality with a small amount of excess reactivity and ascension to full power output shortly after initial reactor startup. Excess reactivity monotonically increases because of breeding until a predetermined burnup is achieved in a selected number of fuel assemblies. The reactivity increase is compensated by control rods, which are gradually inserted into the core to maintain core criticality.

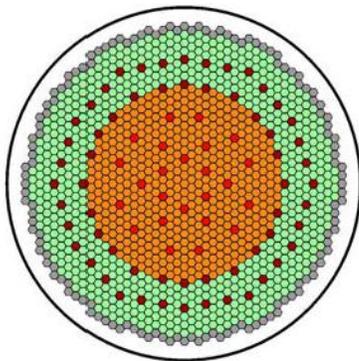


Fig. 2. BOL Core face map (Orange – ACZ, Green FCZ, Red – Movable Control and Safety Assemblies, Brown – FCZ absorber assemblies at EOL, Grey-shield assemblies)

After a predetermined amount of time, the TWR reactor is shut down in order to move high-burnup assemblies to the Fixed Control Zone near the core periphery replacing them with depleted uranium

assemblies. This “fuel shuffling” operation is expected to take one to two weeks depending on the number fuel assemblies requiring shuffling. Fuel shuffling accomplishes three important functions. First, it provides a means of controlling the power distribution and burnup so that core materials remain within safe operating limits. Second, it manages the excess reactivity in conjunction with the control rods. Third, it greatly extends the life of the reactor core because core life is largely determined by the number of depleted uranium assemblies available for shuffling. Fuel shuffling does not involve opening the reactor because all shuffling operations are conducted with equipment installed in the reactor vessel and it occurs at about the same interval for the life of the core. In order to determine what the optimal shuffling patterns for the core are, fuel management computational tools (described in the next section) will be used in conjunction with selected operational information from the core system including neutron flux data, ACZ assembly outlet temperatures and ACZ assembly flow measurements. Data from thermocouples, flowmeters, and neutron flux detectors will serve for verification of fuel management computations and for the adjustments of computational parametric data to match actual measured data.

The large power differences between the fissile assemblies in the ACZ and fertile assemblies in the FCZ require significant differences in assembly flow distribution to match flow to power and thus outlet temperature. This is accomplished through a combination of fixed and variable orifices that make it possible to optimize primary coolant flow proportionally to predicted assembly power. Fixed orifices are installed in assembly receptacles below the core, which mate with seats in the core support grid plate and contain sockets where assemblies are inserted. Each receptacle has orifices, divided in groups to match flow to power generated in the fuel assemblies. The receptacles under the FCZ have very high-pressure-drop orifices to minimize the flow into very low-power fertile assemblies. On the other hand, the receptacles below the ACZ assemblies are divided into several groups of orifices ranging from very low resistance to higher resistance to match the radial power profile in the ACZ. In addition to fixed orifices, each assembly will have the ability to adjust assembly flow by rotation during fuel shuffling operations to enable minor flow adjustments at the assembly level, if needed.

The core system includes movable control elements, placed in the active control zone, which are capable of compensating for the reactivity increase during operation as well as safely shutting down the reactor at any time with appropriate margin for

malfunctions, such as a stuck rod. In addition to limitations against fast withdrawal, the control rod drive mechanisms also use diverse design to minimize the probability of failure. The core FCZ is equipped with a number of absorber assemblies to ensure that the fuel assemblies, which were moved from the ACZ into the FCZ, do not produce excessive power from bred-in fissile material. Absorber assemblies in the FCZ maintain this portion of the core at a very low power and prevent further burnup accumulation, as well as total reactor power increase. The absorber assemblies are mechanically and thermal-hydraulically compatible with fuel assemblies and can take any position within the FCZ. At the beginning of life, they are placed near the core periphery to maximize breeding of fissile material at the ACZ-FCZ interface while at the end of life they are moved closer to the ACZ (shown in Figure 2) to keep the power of discharged fuel assemblies that were moved to FCZ from accumulating more burnup.

One of the challenges in fast reactor design is the short lifetime of boron carbide control rods which is caused by both the excessive swelling from helium generation and the high loss rate of reactivity worth due to depletion of  $B^{10}$ . This challenge is overcome in TWRs by the use of hafnium hydride control rods, which offer up to five times longer lifetime and have a very small reduction of reactivity worth with irradiation because the higher isotopes of hafnium also have significant neutron absorption cross sections. The development of these rods is currently underway in Japan.<sup>26</sup> A row of control assemblies placed on the core periphery serves as both a set of spare control assemblies and a radial shield for the core barrel/reactor vessel wall. The spare rods are within the reach of an offset arm In-Vessel Handling Machine (IVHM) and have handling sockets to enable their movement by the IVHM and replacement of control rods that reached their end of life.

Reactor safety considerations for TWRs are quite different from LWRs. Loss of primary coolant accidents are not credible in pool-type liquid metal reactors employing a containment vessel and thus one of the most challenging design basis accidents for LWRs is non-existent in TWRs. Furthermore, the large thermal inertia and high boiling point of the primary sodium pool make the time evolution of thermal transients much slower in TWR compared to LWRs. This slow time evolution of transients makes it possible to design a core that can achieve reactor shutdown through net negative reactivity feedbacks and remove the decay heat by inherent means, such as natural circulation of coolant without the need for emergency diesel powered safety grade pumps.

Loss of primary coolant flow and loss of heat removal do present a design basis challenge to TWR just as they do in LWRs. However, intrinsic features of the core design with metal fuel causes the collective effect of temperature coefficients of reactivity to be negative at the beginning of life. This is because to achieve inherent shutdown without scram, fuel temperature has to decrease as fission power is reduced to zero, resulting in a reactivity addition because of negative fuel temperature feedback. This reactivity increase is more than compensated by reactivity reduction from coolant temperature increase, primarily due to a negative core radial thermal expansion coefficient. Metallic fuel, which has a small negative fuel temperature feedback and thus a small positive reactivity addition in transients without scram, in combination with a large heat storage capacity of the pool design, makes it possible to design a sodium cooled core that achieves inherent shutdown without exceeding safe temperature limits on cladding and fuel. These characteristics were shown by Wade et al.<sup>27,28</sup> and confirmed by tests in Experimental Breeder Reactor II (EBR-II).

The TWR core is designed using these principles such that safe core cooling is achieved even in the event that the scram system fails to shutdown the reactor. The ability to survive Anticipated Transients Without Scram (ATWS) surpasses the NRC regulatory requirements for light water reactors. TWR core designers expect that satisfactory ATWS response will be achieved and are attempting to ensure that not only will the TWR survive this extremely unlikely event, but that the ATWS event will have minimal impact on the core lifetime – a feat that cannot be assured for LWRs. Initial calculations have confirmed that the TWR core indeed exhibits this attractive feature at the beginning of life.

### *II.C. Modeling and Simulation*

In order to provide independent checks as well as to trade off accuracy and computer time, TerraPower is using Monte-Carlo and deterministic simulation tools based on both MCNPX and REBUS.

Monte-Carlo was chosen as the baseline high fidelity transport method because it can represent the neutron distribution in space, energy and angle with essentially infinite resolution and without the need to specify and validate various binning approximations in all those dimensions. The most notable deficiency of the standard Monte-Carlo method is its computationally intensive nature. For this reason, deterministic methods in REBUS were used for most of the optimization and sensitivity studies.

TerraPower is using MCNPX version 2.6c<sup>29</sup> with ENDF/B-VII cross section data.<sup>30</sup> MCNPX had already coupled the Monte-Carlo neutron transport to the CINDER90 transmutation code<sup>31</sup> using a second order Runge-Kutta method. In each sub-step of the Runge-Kutta method, the Monte-Carlo solves for the steady-state neutron distribution using the spatially dependent nuclide distribution evolved by CINDER90. This neutron distribution, normalized to a specified power level, is then used by CINDER90 to perform the nuclide transformations.

CINDER90 uses decay chains to couple and evolve 3400 nuclides with an internal database of neutron cross-sections and decay rates. In the absence of neutrons this is a straightforward method that uses exponentials to handle any combination of time step and decay rates, but neutron absorption forms loops in these decay chains which must be iterated to achieve a given accuracy. For high burnup TWRs it was found that mass conservation was not adequate and that fixes had to be applied to the chain loop termination conditions. To be assured that CINDER90 was now evolving nuclides accurately two other methods of solving the transmutation equations were implemented: ExpoKit,<sup>32</sup> a Krylov subspace projection method of computing matrix exponentials, and a direct linear matrix solution. The very fast decay rates were slowed down in order to get ExpoKit to converge and the linear matrix method required very small time steps for accuracy. Neither of these are a good general purpose method but they did confirm that the modified CINDER90 package was performing accurately.

The high burnup of TWRs has also required improvement in methods of communicating properties of the ~1300 CINDER90 fission products to the 12 that can be efficiently handled in the Monte-Carlo transport part of the simulation. By comparing calculations using 12 and 213 fission products it was found that simply ignoring others is not adequate, but that scaling the amount of each of the 12 fission products to account for the neutron absorption of its ignored neighbors produced good results as shown in Figure 3.

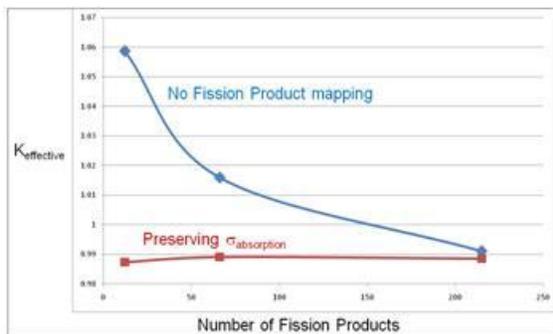


Fig. 3. Criticality of a simplified high burnup system as a function of the number of fission products kept in the Monte-Carlo transport simulation. Mapping the Cinder90 fission products onto those kept in a way that preserves their macroscopic absorption cross-section allows most scoping calculations to run with keeping only 12 fission products in the transport calculation.

In some TWR designs, the placement of control rods is used to shape and drive the burn wave. To simulate this in MCNPX an automated control process was implemented that distributes control according to some desired shape and in a way that automatically maintains criticality. The most realistic of these methods inserts a specified control material at a finite number of control rod positions specified in the problem definition.

Other TWR designs have fuel assemblies which are periodically moved from one location to another in order to achieve adequate breeding of fissile actinides while also minimizing the neutron induced damage to structural materials. High-level adaptive fuel management routines were added to MCNPX to model these movements.

Release of fission product gasses is simulated as part of the transmutation process by including an additional "decay" branch in the reaction chain. In this way, short lived gasses naturally deposit their daughters at the fission site while long lived gasses may be removed to the plenum before they decay. The fission gas removal rate is a function of burnup and temperature history and is supplied by separate fuel evolution calculations.

Typical TWR simulations are extremely computationally intensive because they employ 20,000 to 40,000 regions, each of whose compositions are separately burned and tracked. Running just one 50-year simulation on a single computer core would take more than a month. As a result of this, the address space of the depletion code (Cinder90) was separated from that of the transport code and both the burnup and transport parts of the problem were parallelized. Designers now typically run with 128 cores per problem which reduces the turnaround time on a complex design test to about 8 hours.

The Message Passing Interface (MPI) and memory usage for the neutron transport part of the simulations have also been optimized to the point where even all 1,104 cores in the TerraPower computer cluster can be efficiently used on a single problem. Single, accurate,  $k$ -effective (reactivity) measurements that would have taken 3 days to run on

a single core, can be run on TerraPower's compute cluster in a few minutes. Figure 4 shows how the time to run a computation depends nearly inversely on the number of computer cores applied to the problem.

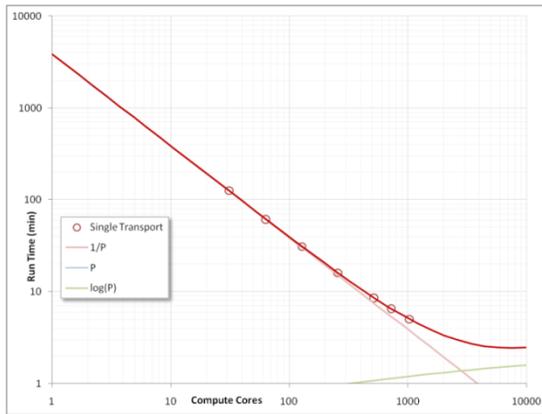


Fig. 4. The run time of a high-precision neutron transport problem as it depends on the number of compute cores applied. The red line through the points shows an estimate of how the curve might continue beyond 1,024 cores.

In a parallel effort, the REBUS-PC 1.04 and MC<sup>2</sup>-2 codes have been put to use for objective-function evaluation in a massively parallel fuel management optimization suite. Controller software searches through various fuel movements and evaluates each perturbation in a parallel manner. When all simulations finish, the controller decides which particular movement is preferred and proceeds with the next cycle.

To understand the behavior of a complex reactor core, easy-to-use 3D visualization is essential. Issues such as problem specification errors, power and neutron flux distributions, and materials damage measurements are more easily understood when a designer can step smoothly, in space and time, through a 3D visual model of the reactor with color coded indicators. TerraPower has developed a data viewer program called XTVIEW (shown in Figure 5), which displays simulation results retrieved from a specialized database.

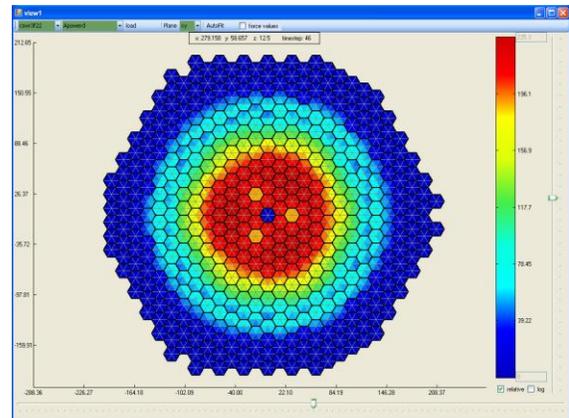


Fig. 5. XTVIEW - TerraPower's 3D visualization tool. Slider controls allow the user to scan through the reactor, in space and time, to understand its performance.

#### II.D. Economic Case for TWRs

The competitiveness of the TWR is of paramount importance because global adoption of TWR reactors likely will be driven in large part by economic advantage. In the near-to-intermediate term, nuclear systems being deployed will be based on light water reactor (LWR) technology. Since the market share for nuclear power can be expected to grow based on electricity consumption growth and global climate change considerations, the TWR will have to compete with LWR plants to be the nuclear technology of choice.

To make this assessment, TerraPower has developed a self-consistent Technology Development and Deployment Plan. Together with the associated cost and revenue projections, the plan is used to project program rates of return and the levelized cost of electricity for a TWR. Together, the revenue, cost, and schedule information is used to analyze investment returns as well as evaluating sensitivities to changes in input parameters.

Where is the TWR economic advantage compared to an LWR? Here we will focus on one example, fuel. A 1-GWe LWR requires an enriched first core, followed by enriched fresh reload fuel for a third of the core about every 18 months. The comparable TWR requires an initial core load that in the early TWRs may contain on the order of two times as much fissile material as an LWR first core. However, because the TWR core lifetime can be achieved using only the initial fuel load, no reloads would be needed. Even based on the present value of the avoided reloads, the TWR would enjoy a fuel cost advantage of several hundred million dollars. The additional fuel advantage derived economic benefits

such as the insensitivity to fluctuations in enrichment and uranium prices over time are not included here.

In another example, the TWR is a sodium cooled, fast reactor that would operate at higher temperatures than an LWR. As a result, for an LWR and TWR of comparable thermal powers, the TWR would operate at higher efficiency and produce about 20% more electrical power. In the 1-GWe power range, this additional 200 MWe represents an increased revenue of over \$100 million annually.

As a final example, TWR waste costs would be reduced. Whether it is ultimately on-site or repository storage, due to higher density fuel, higher efficiency, and higher burn-up operation, the mass of TWR spent fuel would be substantially reduced and lead to additional cost advantages. There are other examples of TWR advantages that remain to be analyzed and their economic impacts quantified such as the significant savings from the elimination of the need for reprocessing facilities and a reduced need for enrichment facilities (eventually not needing any) on the national nuclear energy program level. From a longer-term energy security perspective, the TWR fuel cycle, without reprocessing and enrichment, is expected to exhibit significant savings when compared to closed fuel cycles currently envisioned. In all of these cases our work to date indicates that major advantages of the TWR support engineering embodiments that will make sound economic sense.

### *II.E. Repurposing "Used" TWR Fuel*

The TWR is designed to be as neutronicly efficient as possible to permit operation at lower peak fluences and allow construction using presently available materials. One consequence of this neutronic efficiency is that it allows fuel criticality to be maintained over a much longer range of burnup and fluence. From our calculations, fuel bred in a TWR is able to stay critical to burnup fractions of over 40%, well past the average burnup of approximately 15% achieved in a first generation TWR. As a result, used TWR fuel is well suited to recycling via fuel recladding, a process in which the old clad is removed and the used fuel is refabricated into new fuel. This process produces usable fissile fuel without the proliferation risk of fissile material separations.

The idea of fuel recycling through thermal and physical processes is not new; it was originally part of the EBR-II Fuel Cycle Facility.<sup>33</sup> In this process, the used fuel assemblies are disassembled into individual fuel rods which then had their cladding mechanically cut away. The used fuel then undergoes a high temperature (1300-1400° C) melt refining process in an inert atmosphere which separates many

of the fission products from the fuel in two main ways; the volatile and gaseous fission products (e.g., Br, Kr, Rb, Cd, I, Xe, Cs) simply escape while the more than 95% of the chemically-reactive fission products (e.g., Sr, Y, Te, Ba, and rare earths) become oxidized in a reaction with the zirconia crucible and are readily separated. The melt-refined fuel can then be cast or extruded into new fuel slugs, placed into new cladding with a sodium bond, and integrated into new fuel assemblies. The used cladding and separated fission product waste from the process can be safely stored without proliferation risk, and are modest in mass and volume.

Fuel recladding accomplishes several things: first, the fuel lifetime is enhanced by the removal of gas bubbles and open porosity which causes swelling and leads to stresses between the fuel and cladding. Second, new cladding can be expected to endure a much higher fluence than will already-irradiated cladding. Third, the removal of a large fraction of fission products improves the reactivity and 'neutronic longevity' of the fuel along attainable fractional burnup lines, since parasitic absorptions in fission products are substantially reduced. Finally, since the isotopic and chemical-elemental compositions of a fuel pin have a strong axial dependence due to neutron fluence flux gradient, the opportunity would allow one to axially segment each pin, or pins as a group, prior to melting, and to thereby realize a set of purified melts of markedly distinct isotopic and chemical compositions. Each of these different melts may be dispatched to entirely new fuel pins or to particular axial segments of new pins, thereby providing cast-in isotopic-&-chemical structure for the new pins and fuel assemblies.

TWRs are presently designed to discharge their fuel at an average burnup of approximately 15% of initial heavy metal atoms, with axial peaking making the peak burnup in the range of 28-32%. Meanwhile, as the calculations in Figure 6 show, feed fuel bred in a TWR of nominal 'smear' composition remains critical to over 40% average burnup, even without any fission product removal via melt refining. Including the effect of periodic melt refining allows burn-ups exceeding 50% to be achieved. Therefore, fuel discharged from a first generation TWR still has most of its potential life remaining from a neutronic standpoint (even before the "life extension" associated with thermal removal of fission products during recladding is considered) and would be available for reuse without any need for fissile separations.

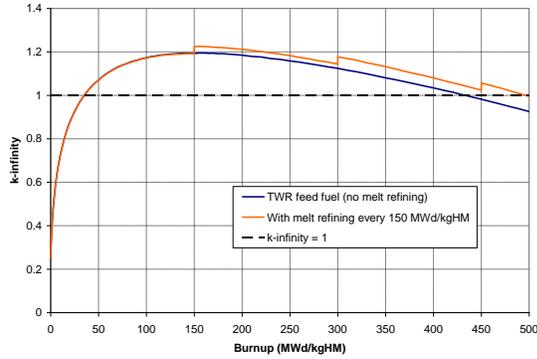


Fig. 6.  $k$ -infinity evolution for a representative TWR fuel-load.

Due to the tremendous neutronic margin available, a unit of fully burned TWR fuel generates enough excess neutrons to breed more than 3 units of fresh TWR feed until each of these is critical (based on full core calculations). In other words, TWR feed fuel can potentially multiply itself by a factor of three within each fuel generation, or substantially more if some fission products are removed via at least one melt-refining process. This multiplication can be carried out either at the end of life of the first TWR, when the fuel from the TWR core would be removed, reclassified and used as driver fuel to start-up 3-4 new TWR cores, or during the TWR core life, since each 15-20 years of TWR operation produces enough fuel to start a new TWR core of the same power rating. Meanwhile, at the 15% average discharge burnup attainable with a first generation TWR, there is relatively little neutronic margin between what is required to keep a reactor core-load critical and what is needed for self-propagating breed-burn operation. Also, the axial peaking in TWR fuel assemblies means that the low-burnup axial ends have been bred into critical fuel, but haven't been used any further. Therefore while a first generation TWR burning and breeding wave can propagate indefinitely, it is able to grow radially only gradually over time.

Recladding changes this picture by raising the maximum burnup achievable with TWR fuel, and furthermore by allowing the axial disassembly of fuel so that the entire length of each TWR fuel assembly can be fully used. For example, used TWR fuel can be refabricated into new fissile fuel and repurposed for a variety of applications, including use in small modular reactors of intrinsically poorer neutron economies or as "seed fuel" for starting unenriched core-loads of subsequent TWR generations. This latter application of reapplying used fuel to start up subsequent TWRs is particularly interesting since the factor of three multiplication per fuel generation is

capable of supporting indefinitely-great TWR build-outs without any enrichment or fissile material separations.

Recladding also presents an additional opportunity for TWR used fuel by allowing future advances in cladding and material technology to be applied to TWR fuel. As fuel leaves an  $N^{\text{th}}$  generation TWR, recladding gives it an opportunity to be reemployed using next generation technology, which may enable higher burn-ups and fluences, higher temperatures, or higher power densities to be achieved. Provided cladding materials can be developed to perform reliably past maximum fuel burn-up limits, then recladding facilities will no longer be needed.

### II.F. Plutonium Vector Degradation

The unique configuration of a TWR allows its fuel to maintain its criticality over a higher burnup and fluence than typical fast reactor configurations. The ability of TWRs to deeply burn their fuel means that the isotopic composition of any resulting plutonium can be deeply degraded, to the extent that discharged TWR fuel has a plutonium vector comparable to that of highly proliferation resistant spent LWR fuel. The ability of a TWR to achieve this feat without the use of reprocessing to chemically separate plutonium is unique among fast reactors.

Several key features make the TWR distinctive. For example, its fuel elements are designed to minimize parasitic losses and spectral softening. This is accomplished by having a high fuel volume fraction and minimizing the relative amount of coolant, structure, and alloying materials. Another key feature is that the burning region in a TWR is surrounded by subcritical feed fuel, consisting of natural or depleted uranium, which absorbs leakage neutrons from the burning region and uses them to breed new fuel. Past a certain thickness of feed fuel surrounding the core of approximately 70 cm (or about 5 assembly rows) the fraction of neutrons leaking from a TWR is effectively zero. These neutron conserving features accomplish two things: first, they minimize the burnup and fluence required to achieve wave propagation which eases material degradation issues and enables the creation of a TWR with existing materials. Second, they increase the maximum burnup and fluence the fuel can sustain before the accumulation of fission products makes the fuel subcritical.

This second point is illustrated in Figure 7. It compares the reactivity evolution of TWR feed fuel and enriched fuel from a typical sodium fast reactor which is modeled as having SuperPhénix fuel, coolant and structure volume fractions with 75%

smear density and an initial enrichment of 16%. Typical sodium fast reactor fuel must start at a high enrichment to achieve criticality and all the excess reactivity of fresh fuel is lost to control elements and leakage from the core. The fuel quickly loses reactivity as  $U^{235}$  is depleted, and becomes subcritical at approximately 310 MWd/kgHM burnup. At the point where the fuel becomes subcritical, about half of the total fissions are due to  $U^{235}$ , and the utilization fraction of  $U^{238}$  is less than 20%. Meanwhile TWR feed begins as subcritical fertile fuel, consisting of either depleted or natural uranium, and gains reactivity as  $Pu^{239}$  is bred in. Once the fuel becomes critical, excess reactivity is offset by breeding additional subcritical feed fuel (during the first 50 MWd/kgHM of burn-up, the driver fuel makes the reactor critical). A total fuel burnup of over 400 MWd/kgHM can be achieved before the fuel becomes subcritical, and since the fuel begins as nearly all  $U^{238}$ , the  $U^{238}$  utilization fraction is over 40%.

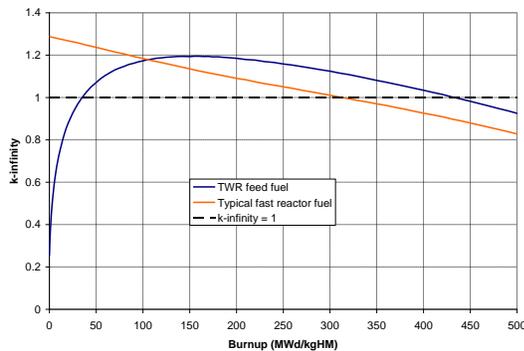


Fig. 7. k-infinity evolution for TWR and typical fast reactor fuels.

The importance of  $U^{238}$  utilization is illustrated in Figure 8 which shows the plutonium isotope evolution as a function of  $U^{238}$  utilization in a TWR spectrum. The curves are representative of the plutonium vector evolution in fast reactors. At low utilization, the plutonium produced is essentially all  $Pu^{239}$ , since one begins with  $U^{238}$  and no plutonium. At higher utilizations, the plutonium quality becomes increasingly degraded as higher isotopes of plutonium are created. At the point which TWR feed fuel's k-infinity falls below unity, the fissile Pu fraction is under 70%, similar to reactor-grade plutonium from LWR spent fuel. Additionally, the plutonium in TWR spent fuel is contaminated to a much higher degree with fission products, making it more difficult to handle and reprocess without needed infrastructure, and therefore less attractive as a target for diversion.

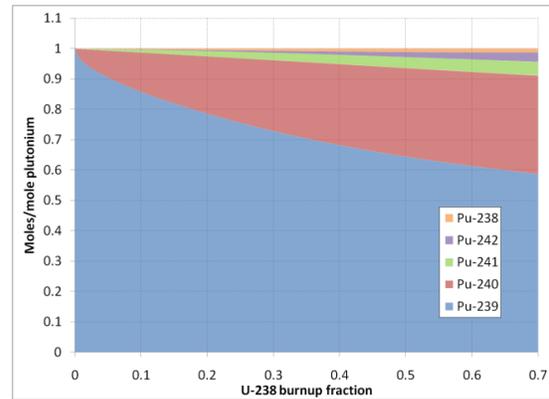


Fig. 8. Fast reactor plutonium vector evolution.

### III. CONCLUSIONS

TWRs featuring high fuel utilization efficiency offer inexhaustibly renewable and eminently-economic nuclear energy in quantities sufficient for the entire human race. The approximately 40-fold improvement in attainable fuel utilization with well-designed TWRs enables the forever economic recovery of uranium from seawater, an inexhaustible and continuously renewed resource. Uranium derived from seawater also has the notable advantage of being more equitably distributed than terrestrial resources (which include wind and solar energy resources), because all that is needed to make use of it is access to any portion of the world's ocean. Any nation with a TWR based energy infrastructure thus would be able to benefit from tremendous energy security advantages.

A nuclear infrastructure based on TWRs requires no reprocessing capabilities, and eventually no enrichment capabilities, so that it can be established and expanded without provoking either of the two major proliferation concerns associated with traditional nuclear energy infrastructures: weapons-grade uranium diverted out the front-end of the nuclear fuel cycle or weapons-grade plutonium diverted out of its back-end. Both enrichment and reprocessing carry not only large and unavoidable monetary costs, but also statistically imposed security costs, as enrichment and reprocessing plants serving civilian nuclear power needs can also be used to produce materials for both official and clandestine nuclear weapons production. This advanced class of power reactors allows the substantial imposed costs of enrichment and reprocessing to be entirely avoided. Furthermore, TWRs are unique in their ability to offer a sustainable nuclear energy system without requiring any capability for producing weapons materials. A practical elimination of the

risks associated with the two most proliferation prone parts of the nuclear fuel cycle, while producing the same emissions-free nuclear electricity, allows for a clear separation in the international community of those countries pursuing peaceful uses of nuclear energy from those who are not.

Disposal of TWR spent fuel is greatly facilitated by its smaller mass and volume for a given amount of electricity generated, relative to LWRs. The ability of TWRs to use unenriched uranium as fuel also provides great benefits for handling nuclear waste. The remarkably low cost of unenriched fast reactor fuel enables a sizable fuel store to be included in the reactor's sealed core which is large enough to suffice for many decades of full-power-operation. Correspondingly, the 'ashes' of efficiently burned fuel can be kept in this sealed reactor core for decades, without ever requiring special storage, transport, or disposal. During residence of such duration in the reactor, a majority of the high activity fission products originally created in the used fuel would have decayed to stable isotopes, thereby greatly reducing the complexity, and thus cost, of safe disposal. Meanwhile, this intrinsic deferral of fuel disposal for many decades reduces the present value of its eventual disposal cost by at least an order of magnitude, while also enabling one to take advantage of far future disposal technologies and of fully complying with the pertinent safety and security standards of future generations.

Finally, TWRs are poised to become a near-term reality, since they are integrations of already proven reactor technologies and are therefore capable of demonstration and initial deployment on a single decade time scale. Thereafter, due to their exceptionally efficient neutron economics and consequently their high fuel-breeding rates, TWRs offer a potentially large build-out rate commencing within two decades of start-up of the first generation reactor.

#### NOMENCLATURE

"Burn-up" is used to indicate burn-up in atom percent (at%), where 1 at% is equivalent to 9.4 MWd/kgHM.

#### REFERENCES

1. S. M. Feinberg, "Discussion Comment," *Rec. of Proc. Session B-10*, ICP UAE, United Nations, Geneva, Switzerland (1958).
2. M. J. Driscoll, B. Atefi and D. D. Lanning, "An Evaluation of the Breed/Burn Fast Reactor Concept," MITNE-229 (Dec. 1979).

3. L. P. Feoktistov, "An analysis of a concept of a physically safe reactor," Preprint IAE-4605/4, in Russian (1988).
4. E. Teller, M. Ishikawa, and L. Wood, "Completely Automated Nuclear Power Reactors for Long-Term Operation," *Proc. Of the Frontiers in Physics Symposium*, American Physical Society and the American Association of Physics Teachers Texas Meeting, Lubbock, Texas, United States (1995).
5. H. van Dam, "The Self-stabilizing Criticality Wave Reactor," *Proc. of the Tenth International Conference on Emerging Nuclear Energy Systems (ICENES 2000)*, p. 188, NRG, Petten, Netherlands (2000).
6. S. P. Fomin, A. S. Fomin, Y. P. Mel'nik, V. V. Pilipenko and N. F. Shul'ga, "Safe Fast Reactor Based on the Self-Sustained Regime of Nuclear Burning Wave," *Proc. of Global 2009*, Paper 9456, Paris, France (Sept. 2009).
7. N. Takaki and H. Sekimoto, "Potential of CANDLE Reactor on Sustainable Development and Strengthened Proliferation Resistance," *Progress in Nuclear Energy*, **50**, 114 (2008).
8. J. Gilleland, C. Ahlfeld, D. Dadiomov, R. Hyde, Y. Ishikawa, D. McAlees, J. McWhirter, N. Myhrvold, J. Nuckolls, A. Odedra, K. Weaver, C. Whitmer, L. Wood and G. Zimmerman, "Novel Reactor Designs to Burn Non-Fissile Fuel," *Proc. of the 2008 International Congress on Advances in Nuclear Power Plants (ICAPP 2008)*, ANS, Anaheim, Calif., United States, Paper 8319 (2008).
9. E. Teller, et al, "Completely Automated Nuclear Reactors For Long-Term Operation II", *Proc. ICENES '96* (Obninsk, 1996) and UCRL-JC-122708 Pt 2 (UC LLNL Preprint, 1996).
10. R. Hyde, M. Ishikawa, N. Myhrvold, J. Nuckolls and L. Wood, "Nuclear Fission Power For 21<sup>st</sup> Century Needs: Enabling Technologies For Large-Scale, Low-Risk, Affordable Nuclear Electricity", *Proc. 2<sup>nd</sup> COE-INES2 Symposium on Innovative Nuclear Systems*, Yokohama, Nov. 2006, published in *Progress in Nuclear Energy* **50**, p 82-91 (2008).

11. New York Mercantile Exchange charts, 2009.
12. International Atomic Energy Agency and OECD Nuclear Energy Agency. "Uranium 2007: Resources, Production and Demand (Redbook)." NEA No. 6345. 2008.
13. World Nuclear Association, "Supply of Uranium," web information (2009) [<http://www.world-nuclear.org/info/inf75.html>]
14. World Nuclear Association, "Uranium and Depleted Uranium," web information (2009) [<http://www.world-nuclear.org/info/inf14.html>]
15. World Nuclear Association, "Waste Management in the Nuclear Fuel Cycle: Disposal of used fuel and other HLW," web information (2009) [<http://www.world-nuclear.org/info/inf04.html>]
16. K.S. Deffeyes, "World Uranium Resources [Use of Log-Curves in Estimation]", *Scientific American* **242**, 1 (1980).
17. T.L. Ku, K.G. Knauss, G. Mathieu, "Uranium in Open Ocean: Concentration and Isotopic Composition", *Deep-Sea Research* **24**, (1977).
18. M. Tamada, "Current status of technology for collection of uranium from seawater," 42<sup>nd</sup> Session of the Erice International Seminars on Planetary Emergencies, Erice, Italy (Aug. 2009).
19. The Ux Consulting Company, "UxC Nuclear Fuel Price Indicators," web information (2009) [[http://www.uxc.com/review/uxc\\_Prices.aspx](http://www.uxc.com/review/uxc_Prices.aspx)]
20. B. Cohen, "Breeder Reactors: A Renewable Energy Source," *American Journal of Physics* **51** #1 (Jan. 1983).
21. M.M. Sarin, S. Krishnaswami, B.L.K. Somayajulu, W.S. Moore, "Chemistry of Uranium, Thorium, and Radium Isotopes in the Ganga-Brahmaputra River System: Weathering Processes and Fluxes to the Bay of Bengal", *Geochimica et Cosmochimica Acta*, **54**, #5, 1387 (1990).
22. V. Vance et al., "Erosion And Exhumation In The Himalaya From Cosmogenic Isotope Inventories Of River Sediments," *Earth and Planetary Science Letters* **206**, 273 (2003).
23. A.E. Walter and A.B. Reynolds, "Fast Breeder Reactors", Pergamon Press, (1981).
24. C.E. Boardman et al., "A Description of the S-PRISM Plant", Paper ICONE-8168, Proceedings of ICONE-8, 8<sup>th</sup> International Conference on Nuclear Engineering, Baltimore, USA, (April 2-7, 2000).
25. R.L. Klueh, A.T. Nelson, *Ferritic/martensitic steels for next-generation reactors*, Journal of Nuclear Materials 371 (2007) 37-52.
26. T. Kido et al., "Study of an Innovative Fast Reactor Utilizing Hydride Neutron Absorber: Development of Coating Technique on Cladding Inner Surface", Paper 9194, Proceedings of ICAPP09, Tokyo, Japan, (May 2009).
27. D.C. Wade and Y.I. Chang, "The Integral Fast Reactor Concept: Physics of Operation and Safety", Nuclear Science and Engineering, Vol. 100, pp. 505-524, (1988).
28. D.C. Wade and E.K. Fujita, "Trends Versus Reactor Size of Passive Reactivity Shutdown and Control Performance", Nuclear Science and Engineering, Vol. 103, pp. 182-195, (1989).
29. J.S. Hendricks et al., "MCNPX 2.6.0 Extensions", *Los Alamos National Laboratory report LA-UR-08-2216*, (2008).
30. M.B. Chadwick et al., "ENDF/B-VII.0: Next Generation Evaluated Data Library for Nuclear Science and Technology", *Nuclear Data Sheets*, 107(12):2931-3060, (2006).
31. W.B. Wilson, T.R. England, D.C. George, D.W. Muir, and P.G. Young, "Recent Development of the CINDER'90 Transmutation Code and Data Library for Actinide Transmutation Studies", *Los Alamos National Laboratory report LA-UR-95-2181*, (1995).
32. R.B. Sidje, "ExpoKit: A Software Package for Computing Matrix Exponentials", *ACM Trans. Math. Software*, 24(1):130-156, (1998).
33. Argonne National Laboratory, "Description and Proposed Operation of the Fuel Cycle Facility for the Second Experimental Breeder Reactor (EBR-II)", ANL-6605 Report (1963).