A.E.R.E. GP/R 1807

2 EXTRA COPIES

Regraded by authority of the Declassification Office, A.E.R.E.
DATE AND SIGNATURE APRIL 9th 1957 E. Mullen
UNCLASSIFIED
AND PUBLISHED

A.E.R.E. GP/R 1807
copy 1

Re-graded from DC 5
d-d. February 11th 1957.
E.M.

A.E.R.E. GP/R 1807

ATOMIC ENERGY
RESEARCH ESTABLISHMENT

SOME CRITERIA FOR A USEFUL THERMONUCLEAR REACTOR

by

J. D. LAWSON

HARWELL, BERKS.

1955

UNCLASSIFIED

THIS DOCUMENT IS THE PROPERTY OF THE UNITED KINGDOM
ATOMIC ENERGY AUTHORITY.

~~It contains classified atomic energy information and may be shown only to staff who are specially authorized to have access to this information. The official in possession of the document will be responsible for its safe custody, and when not in use it is to be kept under lock and key.~~

[redacted] UNCLASSIFIED

A.E.R.E. GP/R 1807

THIS DOCUMENT IS THE PROPERTY OF THE
UNITED KINGDOM ATOMIC ENERGY AUTHORITY
~~It contains classified atomic energy information and may be shown only to staff who are specially authorised to have access to this information. The official in possession of the document will be responsible for its safe custody and when not in use it is to be kept under lock and key.~~

SOME CRITERIA FOR A USEFUL THERMONUCLEAR REACTOR

by

J. D. Lawson

ABSTRACT

Calculations of the power balance in thermonuclear reactors operating under various conditions are given. Two classes of reactor are considered; first, self sustaining systems in which the charged reaction products are trapped, and secondly pulsed systems in which all the reaction products escape so that energy must be supplied continuously during the pulse. For the second class of reactor it is found that not only must the temperatur be sufficiently high but also the product of particle density and pulse length must exceed a certain value. Finally some of the assumptions in the analysis are examined in more detail, and possible methods of relaxing the criteria found earlier are considered.

A.E.R.E. HARWELL

December, 1955

/MS

HS.8783

UNCLASSIFIED
[redacted]

UNCLASSIFIED

[redacted]

CONTENTS

UNCLASSIFIED

[redacted]

[redacted]

UNCLASSIFIED.

[redacted]

[redacted]

1. INTRODUCTION

In this report the power balance in thermonuclear reactors is considered and criteria which must be satisfied in a useful reactor are found. A "thermonuclear" reactor may be defined as a fusion reactor in which the velocity distribution of the reacting particles is Maxwellian, so that a true temperature may be defined.

Of the exoergic reactions involving light nuclei, the D-D and T-D reactions are by far the most probable at low energies. Of these the T-D reaction has the higher cross section, but since tritium does not occur naturally it is necessary to use a system in which it can be bred. This may be done by capturing the neutrons emitted in the T-D reaction in Li^6, which then decays into T and He^4.

The reactions of interest are shown in Table I.

TABLE I

Reaction	Q, MeV.	σ_{max} barns	Energy of σ_{max}
$H^2(d,n)\ He^3$	3.3	0.09	2 MeV
$H^2(d,p)\ H^3$	4	0.16	2 Mev
$H^2(t,n)\ He^4$	17.6	5.0	150 keV
$Li^6(n,\alpha)\ H^3$	4.8	1/v law	

2. ENERGY PRODUCTION AND LOSS IN A HOT GAS

The energy release per unit time per unit volume is given by

$$P_R = n_1 n_2 \overline{v\sigma}(T) E \qquad \ldots (1)$$

where n_1 and n_2 are the number densities of the nuclei of the first and second kinds, and $\overline{v\sigma}(T)$ is the product of the relative velocities of the nuclei and the cross section averaged over the velocity distribution corresponding to a temperature T, and E is the energy released by one reaction. If the ions are of the same kind (as in the D-D reaction), $n_1 n_2$ is replaced by $2(n/2)^2 = 1/2\, n^2$. The values of $\overline{v\sigma}(T)$ have been calculated from published values of the cross sections for the D-D and T-D reactions(1) by Thompson(2) and these are shown in Table II. The two D-D reactions (see Table I) are equally probable in the range of energies of interest (below about 100 keV) and the cross sections shown in the table are for one of them.

[redacted]

[redacted]

TABLE II

T (ev)	10^2	10^3	10^4	10^5
T (degrees)	1.2×10^6	1.2×10^7	1.2×10^8	1.2×10^9
$\log_{10} \overline{v\sigma}$, D-D	− 30.9	− 22.1	− 18.2	− 16.8
$\log_{10} \overline{v\sigma}$, T-D	− 30.4	− 20.2	− 16.1	− 15.1

Energy can be lost from the hot gas in two ways, by radiation and by conduction. At temperatures above about 10^6 degrees in hydrogen radiation occurs principally as bremsstrahlung (free-free transitions). The mean free path of such radiation is large (several gm/cm^2) and consequently in a reactor of controllable size virtually all of it would escape (see section 5). The Stefan-Boltzmann T^4 law does not hold under these circumstances, the variation of intensity with temperature can only be found by a detailed study of the radiation process. According to Cillié the power radiated per unit volume is

$$P_B = 1.4 \times 10^{-34} n^2 T^{\frac{1}{2}} \text{ watts/cc.} \qquad \ldots (2)$$

Conduction loss is difficult to treat in a general way, since it depends on the geometry of the system, its density and temperature distribution, and also the wall material. In the analysis which follows the conduction loss is entirely neglected. The criteria established therefore are necessary for a successful reactor, but they are certainly not sufficient.

Various idealized systems will now be analysed. Possible methods of setting up such systems will not however be discussed.

3. SELF SUSTAINING SYSTEMS

In a star the temperature is maintained by transfer of energy from the disintegration products to the body of the star. In a terrestrial reactor of controllable size however the range of the reaction products is large compared with the linear dimensions of the reactor. It does not seem possible to contain neutrons, but it is not inconceivable that the charged particles could be kept in by suitable electric and magnetic fields. Some externally supplied energy would probably be needed to maintain these field so that the system would not be truly self-sustaining; however it might be possible to keep this energy small.

The minimum temperature at which such a system could operate may be found by equating that portion of the reaction energy carried by the charged particles to the radiation loss. This temperature is 3×10^8 degrees for the D-D reaction and 5×10^7 degrees for the T-D reaction. In the D-D system it is only just possible to sustain the reaction, since above 10^8 degrees the reaction rate increases with temperature only slightly faster than the radiation. At 10^9 degrees for example a conduction loss equal to the radiation loss would quench the reaction.

4. SYSTEMS IN WHICH THE DISINTEGRATION PRODUCTS ESCAPE

The more realistic case in which the reaction products are not retained in the gas will now be considered. Since some specific proposals are for pulsed systems we shall consider the following idealised cycle: the gas is heated instantaneously to a temperature T, this temperature is maintained for a time t, after which the gas is allowed to cool. Conduction loss is again neglected, and it is assumed that the energy used to heat the gas and supply the radiation loss is regained as useful heat.

We now define an important parameter R, as the ratio of the energy released in the hot gas to the energy supplied. Now the energy released by the reaction does not heat the gas directly, since we have assumed that the disintegration products escape. Heat is generated in the walls of the apparatus, and this has to be converted to electrical, mechanical or chemical energy before it can be fed back into the gas. If η is the efficiency with which this can be done, then we have the condition for a power producing system

$$\eta \, (R + 1) > 1 \qquad \ldots (3)$$

The maximum value of η is about 1/3, so that R must be greater than 2.

For the pulsed cycle described above we have

$$R = \frac{tP_R}{tP_B + 3nkT} = \frac{P_R/3n^2kT}{P_B/3n^2kT + 1/nt} \qquad \ldots (4)$$

The 3nkT term represents the energy required to heat the gas to a temperature T. Electron binding energies are neglected, but the contribution from electrons is included (this accounts for the factor 3 rather than 3/2).

Since P_R and P_B are both proportional to n^2, R is a function of T and nt. In fig. 1 curves of R against T for various values of nt are shown for the D-D reaction assuming that the tritium formed is also burnt. (In practice the tritium would probably have to be collected and fed back into the system with the deuterium.) The line R = 2 is shown dotted in the figure, and it is seen that for a useful reactor T must exceed 10^8 degrees and nt must exceed 10^{16}. These conditions are very severe. Conditions for a T-D-Li^6 reactor, shown in fig. 2, are easier though still severe. The corresponding values of temperature and nt are $T = 3 \times 10^7$ degrees, $nt = 10^{14}$.

To conclude we emphasise that these conditions, though necessary are far from sufficient. The working cycle which has been assumed is very optimistic.

[redacted]

5. EXAMINATION OF SOME ASSUMPTIONS

The analysis of the preceding section contains both explicit and impl assumptions. Some of these will now be examined in more detail to see whether the severity of the criteria established can in any way be relaxed

It is of course easy to postulate systems in which the velocity distribution of the particle is not Maxwellian. These systems are outsid the scope of this report however, and will not be discussed further. Nev theless it should be remarked that at the lower temperature the values of $\overline{v\sigma}$ are extremely sensitive to the shape of the 'tail' of the distribution.

In the analysis it has been assumed that the ion and electron temperatures are equal. Radiation loss would be less if the electron temperatur were less than the ion temperature, and to examine whether this is possibl we assume that all the energy is given to the ions, and that this is communicated to the electrons which then radiate. An approximate formula for the rate of energy interchange between ion and electron at different temperatures is given by Giovanelli(4). Equating this to the radiation loss leads to the formula

$$(T_i - T_e)/T_e = 5.10^{-10}T_e \qquad \ldots (5)$$

From this it is clear that in the temperature range of interest this difference is small.

Nothing may be gained by using a system in which electrons are at a lower temperature than that given by equation (5). The energy loss in suc a system by transfer to the electrons will always be greater than the energ which would be radiated by the electrons if they were at the temperature defined by equation (5).

If a system containing nuclei only could be set up, the radiation loss would be zero. It is impossible however to obtain a high density system in this way because of space-charge repulsion. The electric field at the surface of a sphere of radius 1 cm. containing 10^{12} particles per cm^3 would be 600 kv/cm. The energy release at 10^9 degrees with a T-D mixture would be less than 1 milliwatt per cm^3. Such a system is clearly useless as a source of power.

We have seen that a reduction of the radiation loss does not seem to b practicable. The effective value of nt might however be increased if at t end of the pulse the gas could be cooled in a reversible manner. If a fraction f of the heat supplied to the gas could be regained as electrical or mechanical energy, the quantity nt in equation (4) would be replaced by nt/(1 - f). Thus if 90% of the energy of the hot gas could be recovered reversibly this would be equivalent to an increase of pulse length by a factor 10.

The assumption that the linear dimensions of the reactor are small compared with the range of the reaction products and the radiated photons will now be examined. A figure of 1 gm/cm^2 for the range of the photons

[redacted]

[redacted]

and of the reaction products is assumed. This is of the right order of magnitude at temperatures of the order of 10^8 degrees, but a factor of 10 or so is immaterial in the present argument. The power from a cubical D-D reactor of side 1% of this value (0.01 gm/cm^2) and volume V cm^3 is roughly $5 \times 10^{12}\ V^{1/3}$ watts at 10^8 degrees. This is certainly greater than could be handled in a controlled reactor, and consequently the assumptions that radiation and charged particles escape is justified.

CONCLUSION

Even with the most optimistic possible assumptions it is evident that the conditions for the operation of a useful thermonuclear reactor are very severe.

ACKNOWLEDGMENT

I wish to acknowledge helpful discussions with many colleagues on the subject matter of this report, particularly Drs. J. E Allen,(for the arguments leading to Eqn. 5), P.C. Thonemann and W. B. Thompson.

REFERENCES

1. A summary of published results is given by Hirst (1953) A.E.R.E. Memo. N/M 64.

2. W. B. Thompson, A.E.R.E. Memo. in course of preparation.

3. Cillié, G., 1932, M.N.R.A.S. 92, 820.

4. Giovanelli, 1949, Phil. Mag., 40, 206.

[redacted]

[redacted]

UNCLASSIFIED

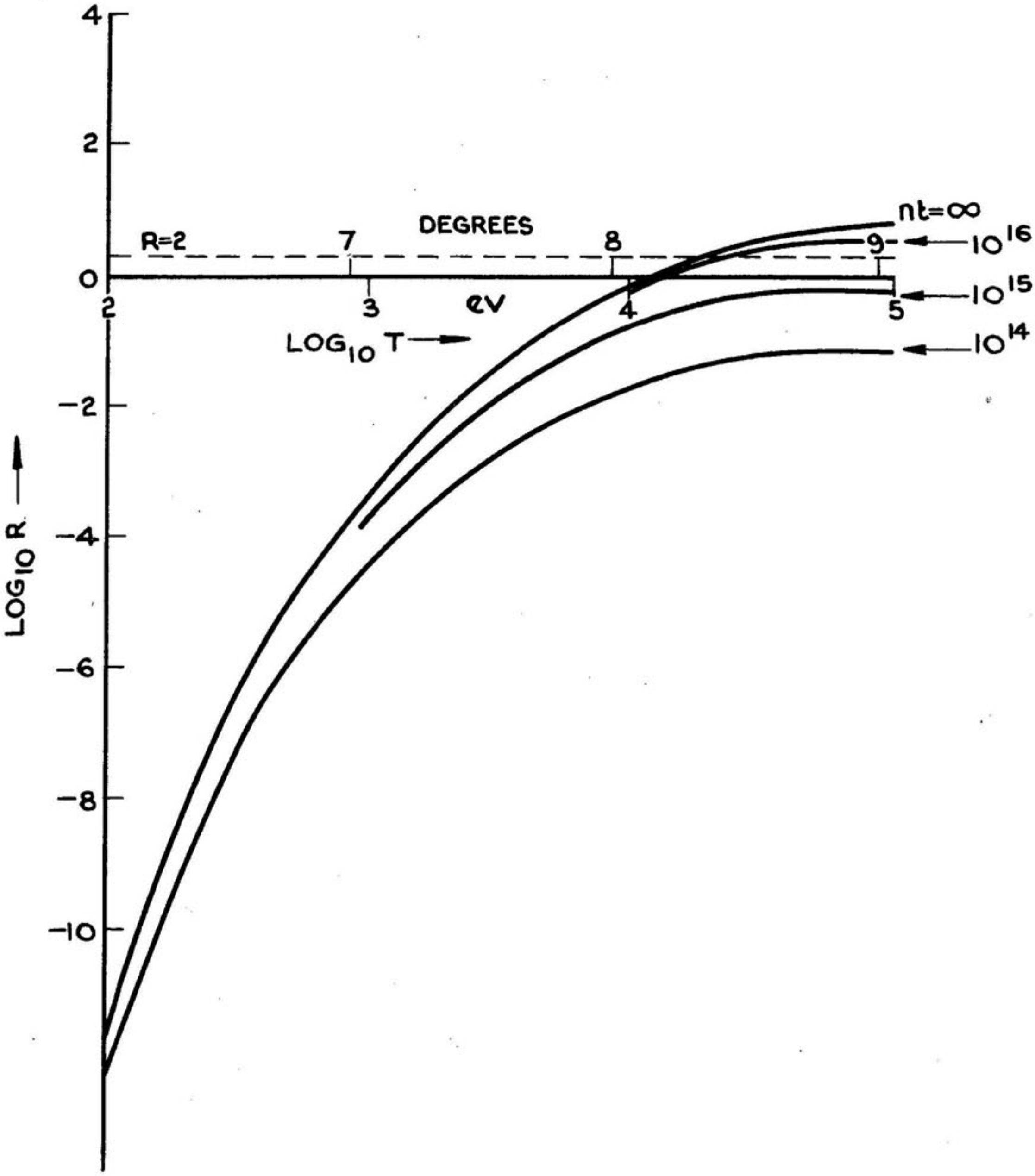
[redacted]

DISTRIBUTION

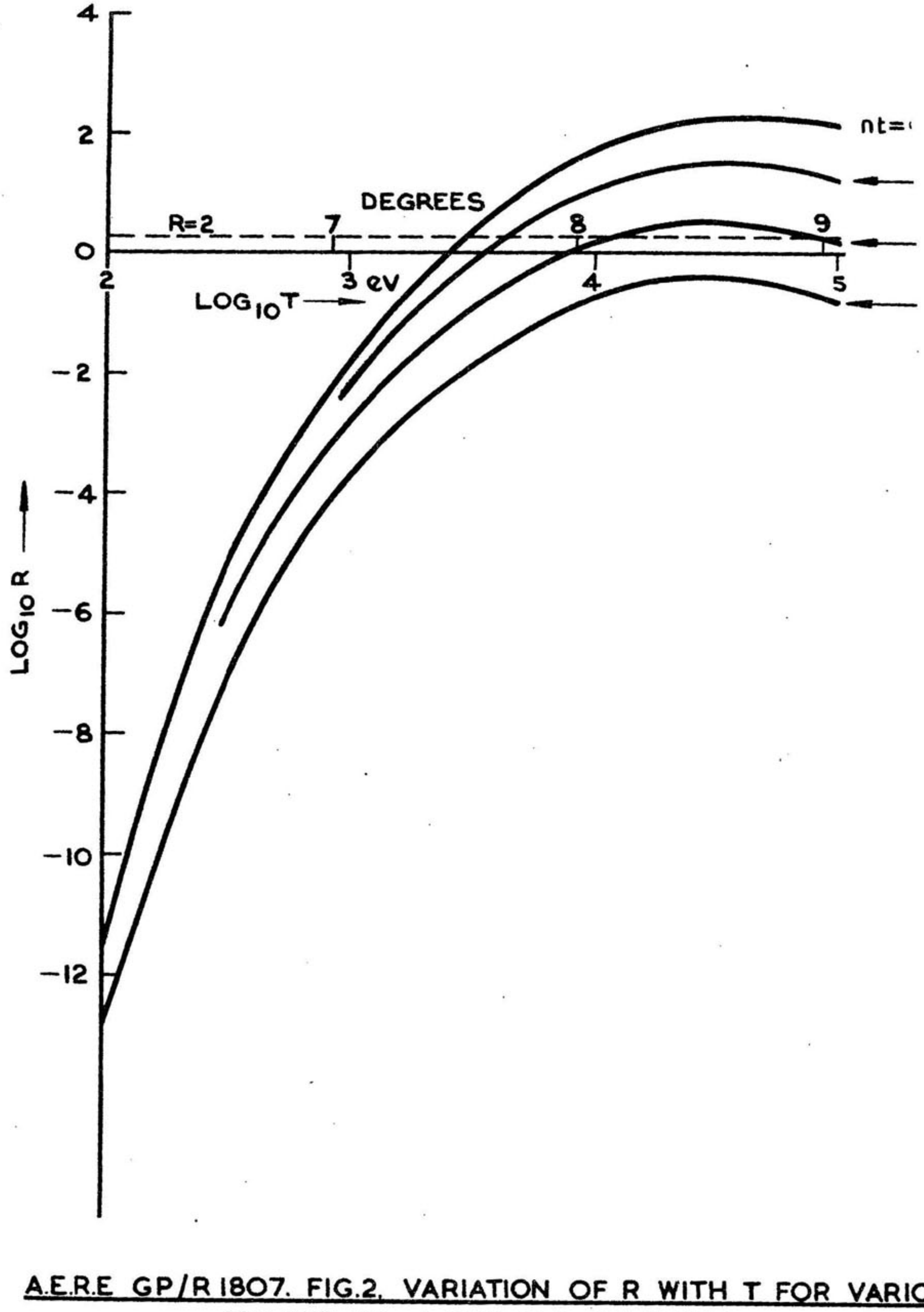
Sir John Cockcroft	A.E.R.E.
Dr. B.F.J. Schonland	"
Dr. E. Bretscher	"
Dr. B.H. Flowers	"
Mr. D. W. Fry	"
Dr. P.C. Thonemann	"
Dr. W. B. Thompson	"
Mr. R. S. Pease	"
Mr. R. Carruthers	"
Dr. R. Hide	"
Mr. W. Millar	"
Dr. J. E. Allen	"
Dr. P. Reynolds	"
Dr. A. P. Willmore	"
Mr. A. H. Turnbull	"
Dr. J. H. Tait	"
Mr. J. D. Lawson	"
Mr. W.R. Cooke	A.W.R.E., Aldermaston
Dr. S. C. Curran	" "
Dr. K. W. Allen	" "
Dr. J. Corner	" "
Sir George Thomson	Corpus Christi College, Cambridge
Dr. T. E. Allibone	A.E.I. Laboratories, Aldermaston
Dr. A.A. Ware	" "
Mr. D. R. Chick	" "

[redacted]

UNCLASSIFIED



A.E.R.E. GP/R 1807. FIG.1. VARIATION OF R WITH T FOR VARIOUS VALUES OF nt FOR D-D REACTION.



A.E.R.E GP/R 1807. FIG.2. VARIATION OF R WITH T FOR VARIOUS VALUES OF nt FOR T-D REACTION.